Department of Mathematics

Tutorial Sheet 6

MTL 106

1 . Aditya and Aayush work independently on a problem in Tutorial Sheet 4 of Probability and Stochastic Processes course. The time for Aditya to complete the problem is exponential distributed with mean 5 minutes. The time for Aayush to complete the problem is exponential distributed with mean 3 minutes.

(a) What is the probability that Aditya finishes the problem before Aayush?

(b) Given that Aditya requires more than 1 minutes, what is the probability that he finishes the problem before Aayush?

(c) What is the probability that one of them finishes the problem a minute or more before the other one?

2 . Let X1 and X2 be two iid random variables each N (0, 1) distributed.

(a) Are X1 + X2 and X1 − X2 independent random variables? Justify.

(b) Obtain E[X1 2 + X22 | X1 + X2 = t].

(c) Calculate E[(X1 + X2 )4 /(X1 − X2 )].

3 . Let X and Y be two identically distributed random variables with Var(X) and Var(Y) exist. Prove or disprove that Var((X+Y)/2) ≤ Var(X)

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4. . Let X and Y be two random variables such that ρ(X, Y)= 0.5. Var(X ) = 1 and Var(Y ) = 4.

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Compute Var(X − 3Y ).

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5 . Let X1, . . . , X5 be i.i.d random variables each having uniform distributions in the interval (0,1). (a) Find the probability that min(X1 , X2 , . . . , X5 ) lies between (1/4, 3/4).

(b) Find the probability that X1 is the minimum and X5 is the maximum among these random variables.

6 . Pick the point (X,Y) uniformly in the triangle {(x, y) | 0 ≤ x ≤ 1 and 0 ≤ y ≤ x}.

Calculate E[(X − Y)2/X]

7 . Find E(Y /x) where (X, Y ) is jointly distributed with density

f(x, y) =

8. Let X have a beta distribution i.e. its pdf is

fX (x) = 1 xa−1 (1 − x)b−1 , 0 < x < 1

β(a,b)

and Y given X = x has binomial distribution with parameters (n, x). Find E(X/y).

9. Consider trinomial trials, where each trial independently results in outcome i with probability 1/3. With

Xi equal to the number of trials that result in outcome i, find E(X1 /X2 > 0).

10. (a) Show that cov(X, Y ) = cov(X, E(Y | X )).

(b) Suppose that, for constants a and b, E(Y | X ) = a + bX . Show that b = cov(X, Y )/Var(X ).

11. Let X be a random variable which is uniformly distributed over the interval (0, 1). Let Y be

chosen from interval (0, X ] according to the pdf:

f(x, y) =

Find E(Y k /X ) and E(Y k ) for any fixed positive integer k.

12. A real function g(x) is non-negative and satisfies the inequality g(x) ≥ b > 0 for all x ≥ a. Prove that

for a random variable X if E(g(X )) exists then